LEVITATION OF SUPERCONDUCTIVE CABLE IN EARTH MAGNETIC FIELD

P. Karban[a)], I. Doležel[b)], B. Ulrych[a)]

[a)]*Faculty of Electrical Engineering UWB, Sady Pětatřicátníků 14, 314 06 Plzeň, E-mail:{ ulrych, karban}@kte.zcu.cz*
[b)]*Institute of Thermomechanics ASCR, Dolejškova 5, 182 00 Praha 8, E-mail: dolezel@iee.cas.cz*

Summary The paper represents an introductory study about a superconductive cable levitating in Earth's magnetic field. Built are two mathematical models of the problem providing both the shape of the arc of the cable and forces acting along it. The theoretical analysis is supplemented with an illustrative example.

1. INTRODUCTION

Earth's magnetic field is produced by rotation of liquid and electrically conductive shell **3** of Earth's core **2** with respect to relatively unmovable shell **1**, see Fig. 1a (another view is in Fig. 1b).

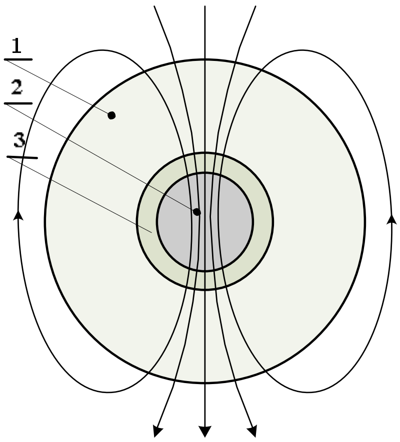

Fig. 1a: Earth magnetic field

This field whose strength on the northern hemisphere reaches values between 38–56 A/m (which depends on the structure of the corresponding lithospheric plates) represents one of very important attributes of the planet Earth. The human population employs this field

- unconsciously – the field represents an "umbrella" protecting Earth's surface against the impact of electrically charged particles arriving to Earth in the consequence of the solar activity and also from space and
- consciously – Earth's magnetic field is used for
  - o orientation on Earth's surface – the first magnetic compasses were built in China, about 600 years before Christ,
  - o geological survey – subsurface deposits of ferromagnetic ores produce local anomalies in Earth's magnetic field,
  - o determination of the time scales in geology and also in archeology.

Nowadays, however, we can often see various considerations aimed at its nontraditional employment. Mentioned can be, for example, „tethers" (particulars can be found in [1] and [2]), which are superconductive cables unwound in a suitable direction from various space objects (sattelites, the last sections of booster rockets, space labs etc.) that can be used as:

- Electric voltage source – a suitably oriented conductor of length l moving at a velocity $\boldsymbol{v}$ in magnetic field of flux density $\boldsymbol{B}$ induces voltage $u_\mathrm{i} = \int_0^l (\boldsymbol{v} \times \boldsymbol{B}) \cdot \mathrm{d}\boldsymbol{l}$ [3]. Transport of such a cable to the orbit is much cheaper than transport of classical photovoltaic cells.
- A suitably oriented conductor of length l carrying current I and moving in magnetic field of flux density $\boldsymbol{B}$ is affected by the Lorentz force of value $\boldsymbol{F}_\mathrm{L} = I\int_0^l (\mathrm{d}\boldsymbol{l} \times \boldsymbol{B})$ [3] that deccelerates or accelerates the space object and shifts it to a lower or higher orbit.

These nontraditional space applications that are intensively studied and now already also experimentally validated (see, for example, [1] – projects *TSS*-1a *Oidipus-C*, NASA+Italian Space Agency) could also be transferred to Earth's surface. The paper deals with one possible application – possibility of employment of Earth's magnetic field for levitation of a superconductive cable closely above Earth's surface, which could be used, for instance, in meteorology, where the levitating cable could replace the trial balloons, particularly for investigation of lower layers of the atmosphere.

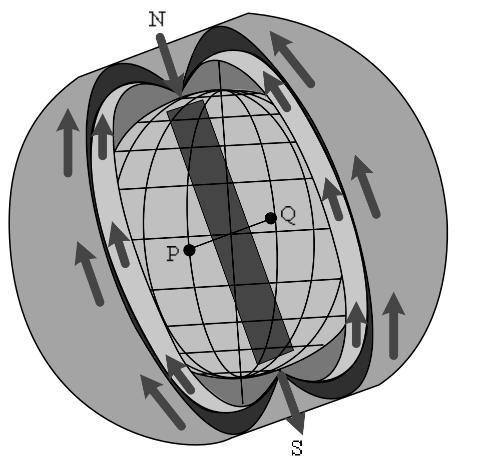

Fig. 1b: Earth magnetic field with the indicated position of the cable (see Figs. 3a, b)

2. FORMULATION OF THE PROBLEM

Fig. 2. depicts an arrangement of a typical superconductive cable for earthly conditions. A similar cable could play a role in the considered case.

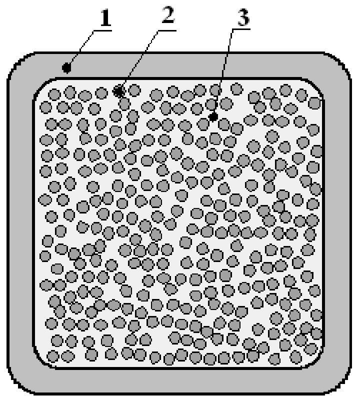

Fig. 2: Arrangement of the superconductive cable

Its superconductive system consists of a greater amount of thin Cu tubes **2** filled in with the actual superconductor and placed in Kevlar shell **1**. The shell representing the carrying element is flown through liquid He **3** that secures the superconductive regime of the cable.

The cable of starting length $s_0 \equiv 2l$ is located on Earth's surface (near the equator) in the horizontal position between points P and Q (see Fig. 1b) whose abscissa is oriented perpendicularly to the force lines of Earth's magnetic field. Near the equator we can consider Earth's magnetic field approximately parallel to Earth's surface, so that its flux density $\boldsymbol{B} = \mathbf{z}_0 B$ (see Fig. 3a).

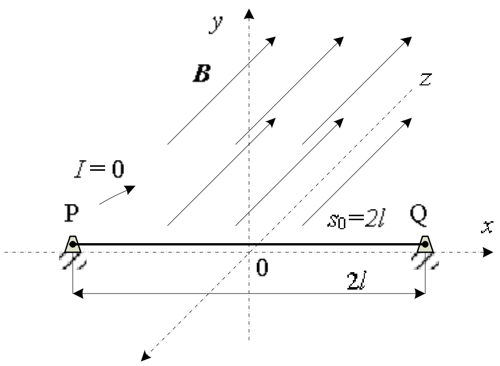

Fig. 3a: The investigated arrangement of a superconducting cable in the domain with uniform flux density – starting position

The points P and Q represent the reel drums equipped with brakes. The cable is wound on them and after their braking off it can freely unwind to a general length $s > 2l$ (Fig. 3b). If the cable carries current I in the indicated direction, it begins to be affected by the total Lorentz force $\boldsymbol{F}_\mathrm{L} = I\int_{-l}^{l}(\mathrm{d}\boldsymbol{l}\times\boldsymbol{B})$ oriented in direction y.

This force then lifts the cable upwards to a general position $y = f(x)$ corresponding to the "braked off" length s of the cable. In this position, however, the specific Lorentz force acting on the unit length of the cable has generally two components $\boldsymbol{f}_\mathrm{L} = \mathbf{x}_0(\pm f_{\mathrm{L},x}) + \mathbf{y}_0 f_{\mathrm{L},y}$. Thus, the levitation effect of Earth's magnetic field can be expected to decrease with growing length s of the cable.

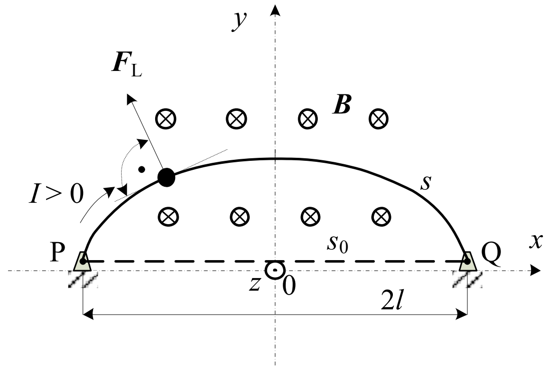

Fig. 3b: The investigated arrangement – final position of the cable

The aim of the paper is to evaluate the

- dependence $y = f(x)$ on the "braked off" length s of the cable and on the value I of the superconductive current,
- dependence of the distribution of forces acting on the cable on the same parameters,
- restriction of the considered levitating effect by the "braked off" length s and current I.

3. MATHEMATICAL MODELS OF THE PROBLEM AND ITS SOLUTION

The mathematical description of the problem may be carried out in two ways:

- Solution of a nonlinear ordinary differential equation formulated as an initial problem and expressing the force and torque balance in an infinitesimal element of the cable.
- Solution of a system of algebraic equations expressing only the balance of forces in a finite element of the cable (the torque balance is here satisfied automatically)

Each of these two models has its advantages and drawbacks. The differential model allows using of numerical algorithms or professionally sophisticated function procedures providing both convergence and required accuracy of solution of the corresponding equation. On the other hand, the algebraic model is more flexible and provides an easier realization of some partial, specific computations such as local distribution of external forces, suppression of deformation of parts of the superconductive cable etc. That is why we used both models that provided a good accordance of the results.

3.1. Differential mathematical model

For obtaining the differential equation we start from Fig. 4a and Fig. 4b showing the lifted cable and situation in its element.

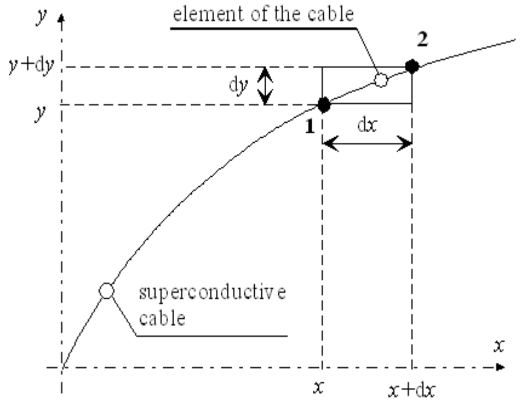

Fig. 4a: The lifted cable with an element

Here H_1, H_2 denote the horizontal components of forces at points **1** and **2**, V_1, V_2 the vertical components and q is the unit weight of the cable.

First let us express the particular components of the Lorentz forces. From Fig. 4b we can easily derive that

$$\mathrm{d}F_{\mathrm{L}x} = -BI\mathrm{d}s \cdot \frac{y'}{\sqrt{1+y'^2}},\ \mathrm{d}F_{\mathrm{L}y} = BI\mathrm{d}s \cdot \frac{1}{\sqrt{1+y'^2}} \quad (1)$$

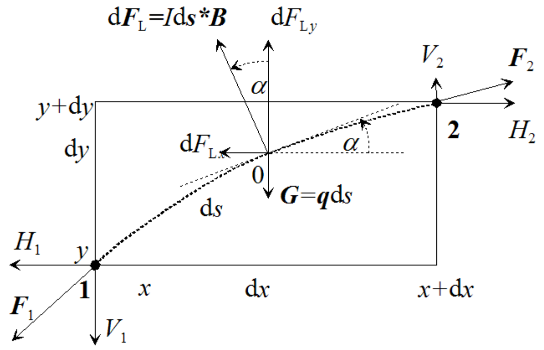

Fig. 4b: Detailed situation in an element between points 1 and 2 (see Fig. 2a)

Here H_1, H_2 denote the horizontal components of forces at points **1** and **2**, V_1, V_2 the vertical components and q is the unit weight of the cable that can generally be quite nonuniform.

First let us express the particular components of the Lorentz forces. From Fig. 4b we can easily derive that

$$\mathrm{d}F_{\mathrm{L}x} = -BI\mathrm{d}s \cdot \frac{y'}{\sqrt{1+y'^2}},\ \mathrm{d}F_{\mathrm{L}y} = BI\mathrm{d}s \cdot \frac{1}{\sqrt{1+y'^2}} \quad (1)$$

and as $\mathrm{d}s = \sqrt{1+y'^2}\,\mathrm{d}x$ we immediately have

$$\mathrm{d}F_{\mathrm{L}x} = -BI\mathrm{d}y,\ \mathrm{d}F_{\mathrm{L}y} = BI\mathrm{d}x\ . \quad (2)$$

The balance equations for the forces in the element read

$$H_1 + \mathrm{d}F_{\mathrm{L}x} = H_2\,,$$
$$V_1 - \mathrm{d}F_{\mathrm{L}y} - q\mathrm{d}s = V_2 \quad (3)$$

and for the torque with respect to point **2**

$$\mathrm{d}F_{\mathrm{L}y} \cdot \frac{\mathrm{d}x}{2} - V_1\mathrm{d}x + \mathrm{d}F_{\mathrm{L}x} \cdot \frac{\mathrm{d}y}{2} = q\mathrm{d}s \cdot \frac{\mathrm{d}x}{2} - H_1\mathrm{d}y\ . \quad (4)$$

As at any point of the cable $V = Hy'$, we immediately have

$$\mathrm{d}V = \mathrm{d}\left(Hy'\right) = H \cdot \mathrm{d}y' + y'\mathrm{d}H \quad (5)$$

where (see (3))

$$\mathrm{d}H = H_2 - H_1 = \mathrm{d}F_{\mathrm{L}x}\,,$$
$$\mathrm{d}V = V_2 - V_1 = q\mathrm{d}s - \mathrm{d}F_{\mathrm{L}y}\,.$$

After substituting from (2) and for $\mathrm{d}s$ we finally have

$$q\sqrt{1+y'^2}\,\mathrm{d}x - BI\mathrm{d}x = H \cdot \mathrm{d}y' + y'BI\mathrm{d}y$$

and hence

$$q\sqrt{1+y'^2} - BI\left(1+y'^2\right) = H \cdot y''\,, \quad (6)$$

which represents a nonlinear differential equation describing the resultant shape of the cable. Force H is given as (see (2))

$$H = BIy + H_0 \quad (7)$$

where H_0 is the horizontal tension in the cable at its beginning ($x = \pm l$). The first initial condition read $y(l) = y(-l) = 0$, the second one is the known length s of the cable.

3.2. Algebraic mathematical model

Consider an arrangement of the superconductive cable in Cartesian coordinate system as is depicted in Fig. 3a. In direction x the distance $l/2$ is divided into N_e uniform elements Δx (Fig. 5a) with an equivalent number of corresponding nonuniform elements Δs_i on the curve $s \approx y(x)$ describing the shape of arc of the cable. To a general distance x_i we assign the general length s_i of the corresponding arc of the cable.

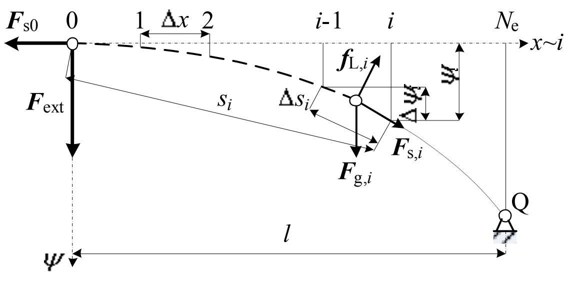

Fig. 5a: Distribution of forces on the cable

The part s_i of the cable is affected by the following forces (Fig. 5b):

- $\boldsymbol{F}_{s0}$, $\boldsymbol{F}_{s,i}$ – internal forces producing tension in the cable,
- $\boldsymbol{f}_{L,i}$ – the Lorentz forces acting perpendicularly on particular elements Δs_i,
- $\boldsymbol{F}_{g,i}$ – weight of particular elements Δs_i,
- $\boldsymbol{F}_{ext}$ – external force loading the cable.

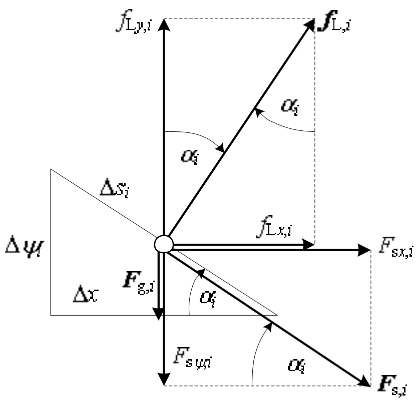

Fig. 5b: Forces in an element of the cable

According to Figs. 5a, 5b we have

$$\begin{gathered}\boldsymbol{F}_{s0} = -\mathbf{x}_0 F_{s0},\ \boldsymbol{F}_{ext} = \boldsymbol{\psi}_0 F_{ext}, \\ F_{s,i} = \mathbf{x}_0 F_{sx,i} + \boldsymbol{\psi}_0 F_{s\psi,i},\ F_{s0} = \left|\boldsymbol{F}_{s0}\right|, \\ F_{ext} = \left|\boldsymbol{F}_{ext}\right|,\ \boldsymbol{F}_{g,i} = \boldsymbol{\psi}_0 F_{g\psi,i},\ F_{g\psi,i} = f_g \Delta s_i, \\ F_{sx,i} = \left|\boldsymbol{F}_{s,i}\right| \cos\alpha_i,\ F_{s\psi,i} = \left|\boldsymbol{F}_{s,i}\right| \sin\alpha_i.\end{gathered} \tag{8}$$

At the same time there holds

$$\begin{gathered}\Delta x = \frac{1}{N_e},\ \Delta s_i = \sqrt{\Delta x^2 + \Delta\psi_i^2}, \\ \sin\alpha_i = \frac{\Delta\psi_i}{\Delta s_i},\ \cos\alpha_i = \frac{\Delta x}{\Delta s_i}.\end{gathered} \tag{9}$$

The conditions of balance of particular forces now read

$$\begin{gathered}x: -F_{s0} + \sum_{k=1}^{i} f_{Lx,k} + F_{sx,i} = 0, \\ \psi:\ F_{ext} - \sum_{k=1}^{i} f_{L\psi,k} + F_{s\psi,i} + \sum_{k=1}^{i} f_{g\psi,k} = 0.\end{gathered} \tag{10}$$

After substitution from (8) and (9) to (10) we obtain

$$\begin{gathered}-F_{s0} + BI\sum_{k=1}^{i} \Delta s_k \sin\alpha_k + F_{s,i}\cos\alpha_i = 0 \Rightarrow \\ F_{s,i} = \frac{F_{s0} - BI\sum_{k=1}^{i} \Delta s_k \sin\alpha_k}{\cos\alpha_i}\end{gathered} \tag{11}$$

Analogously for the components in direction ψ we have

$$F_{ext} - BI\sum_{k=1}^{i} \Delta s_k \cos\alpha_i + f_g \sum_{k=1}^{i} \Delta s_k + F_{S,i}\sin\alpha_i = 0 \Rightarrow$$

$$F_{s,i} = \left(-F_{ext} + BI\sum_{k=1}^{i} \Delta s_k \cos\alpha_i - f_g \sum_{k=1}^{i} \Delta s_k\right) / \sin\alpha_i . \tag{12}$$

Comparison of (11) and (12) provides

$$\frac{F_{s0} - BI\sum_{k=1}^{i} \Delta s_k \sin\alpha_k}{\cos\alpha_i} = \frac{-F_{ext} + BI\sum_{k=1}^{i} \Delta s_k \cos\alpha_i - f_g \sum_{k=1}^{i} \Delta s_k}{\sin\alpha_i} \tag{13}$$

The algorithm of solution of this mathematical model may be realized in the following steps:

- For some value of F_{s0} solution of (13) by, for example, the Regula Falsi method using (9), provides, by means of [4], the value of ψ_i.
- Solution of (11) with respect to (8) provides for the value ψ_i the force $F_{S,i}$.
- The first two steps are repeated for all values of $i = 1, \ldots, N_e$; at the same time we calculate the value of $\sum_{k=1}^{N_e} \Delta s_k$ that is necessary for correction of F_{s0}.
- Condition $\sum_{k=1}^{N_e} \Delta s_k = s$ then provides (again using the Regula Falsi method) the value of F_{s0}.

4. COMPUTER MODEL, ACHIEVED ACCURACY OF SOLUTION

Solution of the mathematical model 3.1 (equation (6)) was carried out by the fourth-order Runge-Kutta method written in Matlab. Proved was very good convergence of the numerical process – for finding of $y(x)$ in the interval $0 \le x \le l = 50$ m with accuracy of 3 valid digits we needed only 50 steps.

Solution of the mathematical model 3.2 was performed by a user program written in Borland Delphi. The convergence of the numerical process for finding both $y(x)$ and $\boldsymbol{F}_S(x)$ was good again. Accuracy of 3 valid digits was reached in about 200 steps.

5. ILLUSTRATIVE EXAMPLE

5.1. Technical specification

Considered is an arrangement of the superconducting levitating cable according to Fig. 2. Its dimensions and all physical parameters are listed in Tab. 1. It is necessary to carry out such a set of testing computations that would provide answers to questions in paragraph 2.

5.2. Results and their discussion

Dependencies $s \equiv y(x)$ on the braked-off length of the cable and current I follow from Fig. 6.

Table 2: Physical parameters of the cable

quantity	symbol	unit	value

distance PQ	$2l$	m	100
length of braked-off cable	s	m	120 130 140
external force	F_{ext}	N	0 (*)
specific weight	f_g	N/m	2.5
Earth's magnetic flux density [6]	$\boldsymbol{B}$	T	$5\cdot10^{-5}$
current in the cable	I	A	$5\cdot10^{5}$ 10^{6} $1.5\cdot10^{6}$

* External force is not considered at this stage of research

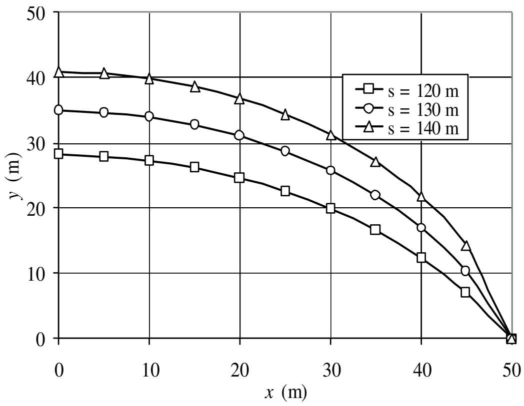

Fig. 6: Dependence of the lift y of the cable on length s $(I = 1.5\cdot10^6 A)$

We can see that the lift increment of the cable decreases with the length s of the braked-off cable. It is obviously caused by the change of orientation of vector $\boldsymbol{f}_{\mathrm{L}}(s)$ along the length s when this length changes, see Figs. 8 and 9. Increase of lift $y(x)$ could be, of course, achieved by diminishment of specific mass $\boldsymbol{f}_{\mathrm{g}}$ of the cable, by growth of current I and, especially, by growth of distance PQ = $2l$.

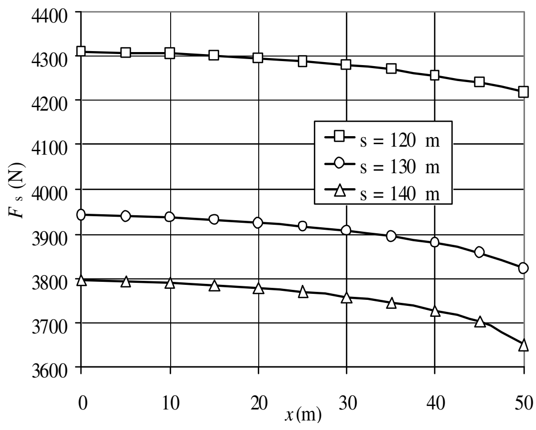

Fig. 7: Dependence of the internal force F_{s} on length s $(I = 1.5\cdot10^6 A)$

Evaluation of distribution of the forces acting on the cable on its braked-off length s and current I can be carried out from Figs. 7, 8 and 9.

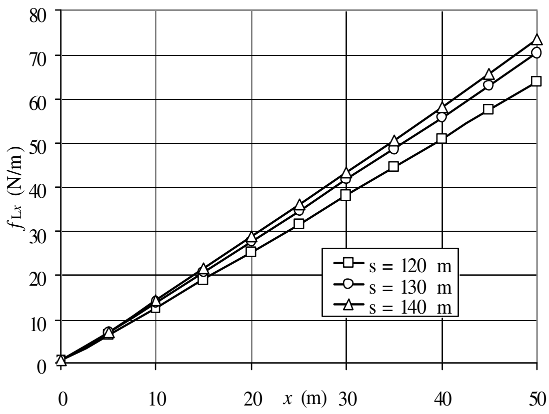

Fig. 8: Dependence of component $f_{\mathrm{L}x}$ of the Lorentz force $\boldsymbol{f}_{\mathrm{L}}$ on length s $(I = 1.5\cdot10^6 A)$

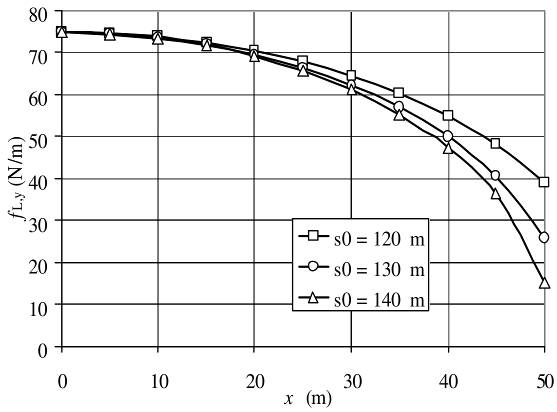

Fig. 9: Dependence of component $f_{\mathrm{L},y}$ of the Lorentz force $\boldsymbol{f}_{\mathrm{L}}$ on length s $(I = 1.5\cdot10^6 A)$

Fig. 7 shows that internal force $F_{\mathrm{s}} = |\boldsymbol{F}_{\mathrm{s}}|$ producing tension in the cable strongly depends on the braked-off length s of the cable. This is evidently associated with the balance of forces acting on the cable. Orientation of the vector of specific force $\boldsymbol{f}_{\mathrm{L,s}}$ figuring in the balance changes with s and depends on its value. But its value is (from the viewpoint of strength of Kevlar whose Young modulus is [5] $E = 1.24\cdot10^{11}$ N/m^2) quite acceptable.

Figs. 8 and 9 show the qualitative and quantitative changes of vector $\boldsymbol{f}_{\mathrm{L,s}}$ produced by tilting of the cable with growth of length s. The vector is always perpendicular to the tangent of curve $\boldsymbol{s} \equiv y(x)$, which results in decrease of the component $f_{\mathrm{L,y}}(s)$, i.e. the levitation component of the total Lorentz force $F_{\mathrm{L,y}}$. This fact represents a certain restriction of levitation effects, but this can be compensated – as said above – by prolonging of the original distance of the cable $2l$.

Restriction of the considered levitation effect by the value of braked-off length s of the cable may be evaluated from Figs. 10, 11 and 12.

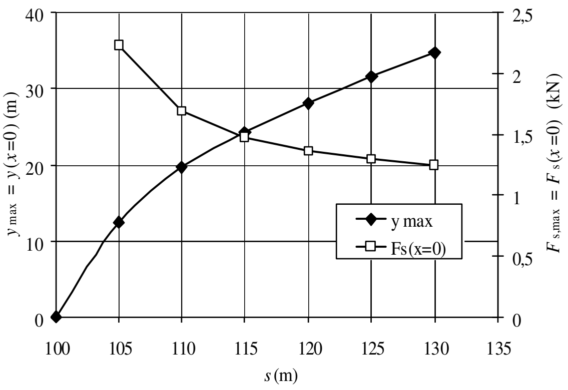

Fig. 10: Dependence of the maximum lift $y_{\max}$ and internal force $F_{s,\max}$ on length s of the cable ($I = 5 \cdot 10^5$ A)

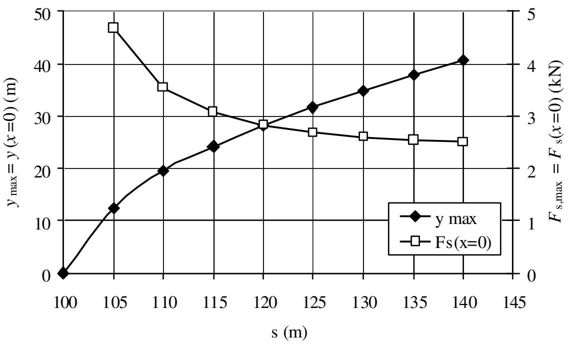

Fig. 11: Dependence of the maximum lift $y_{\max}$ and internal force $F_{s,\max}$ on length s of the cable ($I = 10^6$ A)

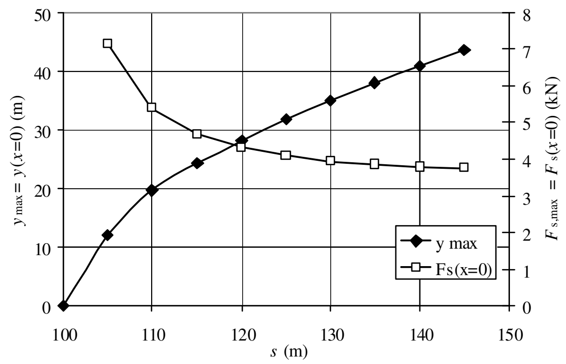

Fig. 12: Dependence of the maximum lift $y_{\max}$ and internal force $F_{s,\max}$ on length s of the cable ($I = 1.5 \cdot 10^6$ A)

The above three figures show that the levitation effect (in the arrangement of parameters in Tab. 1) is limited. From certain value of s (for example for $I = 5 \cdot 10^5$ A from length $s \approx 130$ m) the force $\boldsymbol{F}_s(x = 0)$ stops changing and, therefore, the balance of forces acting on the cable remains the same. Further increase of its length s would lead to increase of its weight, which would break the balance that is the basic condition of successful levitation.

6. CONCLUSION

The paper shows that the idea of a levitating superconductive cable in Earth's magnetic field is, from the viewpoint of the theoretical principles, quite real. Its practical realization would require, however, solution of a number of technological problems (a sufficiently firm and light superconductive cable, easy generation of high current, transport of liquid He to the unwound cable etc.) and also problems of economic character.

Further research of theoretical questions in the domain should be aimed at

- possibilities of increase of the perpendicular component F_{Ly} of the Lorentz force by increase of stiffness of a certain part of the cable,
- solution of the task as a fully 3D problem (respecting of effects of wind etc.).

Acknowledgement

This work has been financially supported from the Grant Agency of the Czech Republic (project No. 102/04/0095).

REFERENCES

[1] Lorenzini, E., Sanmartin, E.: Electrodynamic tethers – ropes in space. Scientific American, July 2005.

[2] Cosmo, M.L., Lorenzini, E.: "Tethers in space, "Handbook, available on address http://cfa-www.harvard.edu/spgroup/handbook.html.

[3] Chari, M.V.K., Salon, S.J.: "Numerical methods in electromagnetism," Academic Press, 2000.

[4] Gerald, C.F., Wheatlley, P.O.: „Applied numerical analysis," Pearson Addison Wesley, NY 2004.

[5] www.azom.com.

[6] Group of authors, „The joy of knowledge: The anatomy of Earth," London 1996, Czech translation Albatros Praha 1995.